Chapter 1

Introduction

In general we do not like to wait. But reduction of the waiting time usually requires extra investments. To decide whether or not to invest, it is important to know the effect of the investment on the waiting time. So we need models and techniques to analyse such situations.

In this course we treat a number of elementary queueing models. Attention is paid to methods for the analysis of these models, and also to applications of queueing models. Important application areas of queueing models are production systems, transportation and stocking systems, communication systems and information processing systems. Queueing models are particularly useful for the design of these system in terms of layout, capacities and control.

In these lectures our attention is restricted to models with one queue. Situations with multiple queues are treated in the course "Networks of queues." More advanced techniques for the exact, approximative and numerical analysis of queueing models are the subject of the course "Algorithmic methods in queueing theory."

The organization is as follows. Chapter 2 first discusses a number of basic concepts and results from probability theory that we will use. The most simple interesting queueing model is treated in chapter 4, and its multi server version is treated in the next chapter. Models with more general service or interarrival time distributions are analysed in the chapters 6, 7 and 8. Some simple variations on these models are discussed in chapter 10. Chapter 9 is devoted to queueing models with priority rules. The last chapter discusses some insentive systems.

The text contains a lot of exercises and the reader is urged to try these exercises. This is really necessary to acquire skills to model and analyse new situations.

1.1 Examples

Below we briefly describe some situations in which queueing is important.

Example 1.1.1 *Supermarket.*
How long do customers have to wait at the checkouts? What happens with the waiting

time during peak-hours? Are there enough checkouts?

Example 1.1.2 *Production system.*
A machine produces different types of products.
What is the production lead time of an order? What is the reduction in the lead time when we have an extra machine? Should we assign priorities to the orders?

Example 1.1.3 *Post office.*
In a post office there are counters specialized in e.g. stamps, packages, financial transactions, etc.
Are there enough counters? Separate queues or one common queue in front of counters with the same specialization?

Example 1.1.4 *Data communication.*
In computer communication networks standard packages called cells are transmitted over links from one switch to the next. In each switch incoming cells can be buffered when the incoming demand exceeds the link capacity. Once the buffer is full incoming cells will be lost.
What is the cell delay at the switches? What is the fraction of cells that will be lost? What is a good size of the buffer?

Example 1.1.5 *Parking place.*
They are going to make a new parking place in front of a super market.
How large should it be?

Example 1.1.6 *Assembly of printed circuit boards.*
Mounting vertical components on printed circuit boards is done in an assembly center consisting of a number of parallel insertion machines. Each machine has a magazine to store components.
What is the production lead time of the printed circuit boards? How should the components necessary for the assembly of printed circuit boards be divided among the machines?

Example 1.1.7 *Call centers of an insurance company.*
Questions by phone, regarding insurance conditions, are handled by a call center. This call center has a team structure, where each team helps customers from a specific region only.
How long do customers have to wait before an operator becomes available? Is the number of incoming telephone lines enough? Are there enough operators? Pooling teams?

Example 1.1.8 *Main frame computer.*
Many cashomats are connected to a big main frame computer handling all financial transactions.
Is the capacity of the main frame computer sufficient? What happens when the use of cashomats increases?

Example 1.1.9 *Toll booths.*
Motorists have to pay toll in order to pass a bridge. Are there enough toll booths?

Example 1.1.10 *Traffic lights.*
How do we have to regulate traffic lights such that the waiting times are acceptable?